Complete reference — Arithmetic · Number Theory · Algebra · Geometry · P&C · Probability · Clocks · Mensuration

ARITHMETIC	FORMULA	NOTE
Percentage Change	`% Change = (New - Old) / Old × 100`	*Multiply by 100 to convert ratio to percentage*
Successive %	`Net% = a + b + ab/100`	*E.g. 10% then 20%: 10 + 20 + (10×20)/100 = 32%*
Simple Interest	`SI = PRT/100  |  A = P(1 + RT/100)`	*P = Principal, R = Rate per annum, T = Time in year*
Compound Interest	`A = P(1 + r/n)^(nt)  |  CI = A - P`	*r = annual rate, n = times compounded per year, t =*
CI — Annual	`A = P(1 + R/100)^n`	*Simplest form when compounded annually*
CI — Half-yearly	`A = P(1 + R/200)^(2n)`	*Rate halved, period doubled*
CI — Quarterly	`A = P(1 + R/400)^(4n)`	*Rate quartered, period quadrupled*
Difference CI-SI	`CI - SI = P(R/100)^2 for 2 years`	*Useful shortcut for 2-year problems*
Time & Work	`1/A + 1/B = 1/T  |  Efficiency ∝ 1/Time`	*A & B together finish in T = AB/(A+B)*
Work Done	`Work = Efficiency × Time`	*If A is twice as efficient as B, A takes half the time*
Pipes — Fill	`Net rate = 1/A + 1/B (both filling)`	*Time to fill = 1/(1/A + 1/B)*
Pipes — Drain	`Net rate = 1/A - 1/B (A fills, B drains)`	*Positive = filling, negative = draining*
Speed-Distance-Time	`D = S × T  |  S = D/T  |  T = D/S`	*Units must be consistent (km/hr or m/s)*
Unit Conversion	`1 km/hr = 5/18 m/s  |  1 m/s = 18/5 km/hr`	*Multiply by 5/18 to convert kmph to mps*
Average Speed	`Avg Speed = 2S1·S2 / (S1 + S2)`	*Only when equal distances at S1 and S2*
Relative Speed — Opp	`Relative Speed = S1 + S2`	*Objects moving towards each other*
Relative Speed — Same	`Relative Speed = |S1 - S2|`	*Objects moving in same direction*
Trains — Cross object	`Time = (L_train + L_object) / Relative Speed`	*L_object = 0 if it is a pole/person*
Trains — Cross train	`Time = (L1 + L2) / (S1 ± S2)`	*+ for opposite, - for same direction*
Boats — Downstream	`Speed = U + V  |  D = (U+V) × T`	*U = speed in still water, V = stream speed*
Boats — Upstream	`Speed = U - V  |  T = D/(U-V)`	*Going against current*
Boats — Still water	`U = (Down + Up)/2  |  V = (Down - Up)/2`	*Find U and V from downstream/upstream speeds*
Alligation	`Cheap : Dear = (Dear - Mean) : (Mean - Cheap)`	*Ratio in which two qualities are mixed*
Mixture Rule	`C1/C2 = (A2 - M)/(M - A1)`	*C1, C2 = quantities; A1, A2 = concentrations; M = m*
Partnership — Simple	`Profit ratio = Capital ratio`	*When time is equal*
Partnership — Compou	`Profit ratio = Capital × Time`	*When time is different*
Ratio & Proportion	`a:b = c:d → ad = bc (product of extremes = means)`	*Cross multiplication rule*
Componendo-Dividendo	`(a+b)/(a-b) = (c+d)/(c-d)`	*Useful for simplifying ratio equations*

NUMBER THEORY	FORMULA	NOTE
HCF × LCM	HCF(a,b) × LCM(a,b) = a × b	*Only valid for exactly two numbers*
HCF of fractions	HCF = HCF of numerators / LCM of denominators	
LCM of fractions	LCM = LCM of numerators / HCF of denominators	
No. of factors	N = a^p·b^q·c^r → factors = (p+1)(q+1)(r+1)	*First prime factorise N completely*
Sum of factors	Sum = [(a^(p+1)-1)/(a-1)] × [(b^(q+1)-1)/(b-1)]...	*For N = a^p·b^q*
Product of factors	Product = N^(f/2) where f = number of factors	
Trailing zeros in n!	Count = floor(n/5) + floor(n/25) + floor(n/125) +	*Each 5 paired with a 2 gives one trailing zero*
Unit digit cycles	2,3,7,8 → cycle of 4 | 4,9 → cycle of 2	*1,5,6 always end in 1,5,6 | 0 ends in 0*
Unit digit of a^n	Find n mod (cycle length), then apply	*E.g. 7^53: 53 mod 4 = 1, so unit digit = 7^1 = 7*
Div by 2	Last digit even (0,2,4,6,8)	
Div by 3	Sum of digits divisible by 3	
Div by 4	Last 2 digits divisible by 4	
Div by 6	Divisible by both 2 and 3	
Div by 8	Last 3 digits divisible by 8	
Div by 9	Sum of digits divisible by 9	
Div by 11	Alternating digit sum (even pos - odd pos) div by	*E.g. 121: 1-2+1=0, divisible by 11*
Fermat's Little Theorem	a^(p-1) ≡ 1 (mod p) for prime p, gcd(a,p)=1	*Used to find remainders of large powers*
Euler's Theorem	a^ϕ(n) ≡ 1 (mod n) when gcd(a,n)=1	*ϕ(n) = Euler's totient function*
Wilson's Theorem	(p-1)! ≡ -1 (mod p) for prime p	
Chinese Remainder	Solve simultaneous modular equations	*x ≡ a (mod m), x ≡ b (mod n) → unique solution mod*
Perfect squares	Last digits can only be: 0,1,4,5,6,9	*Never 2,3,7,8*
Sum 1..n	n(n+1)/2	
Sum of first n odd numb	n^2	*1+3+5+...+(2n-1) = n^2*
Sum of first n even num	n(n+1)	*2+4+6+...+2n = n(n+1)*
Sum of squares 1..n	n(n+1)(2n+1)/6	
Sum of cubes 1..n	[n(n+1)/2]^2	*Always equals (Sum of first n naturals)^2*
Number of primes	Use Sieve of Eratosthenes for small ranges	*Primes < 100: 2,3,5,7,11,13,17,19,23,29,31,37,41,4*
Coprime pairs	gcd(a,b)=1 → a and b are coprime	*Count of numbers < n coprime to n = n·∏(1-1/p)*

ALGEBRA	FORMULA	NOTE
(a+b)^2	a^2 + 2ab + b^2	
(a-b)^2	a^2 - 2ab + b^2	
(a+b)^3	a^3 + 3a^2b + 3ab^2 + b^3	
(a-b)^3	a^3 - 3a^2b + 3ab^2 - b^3	
a^3 + b^3	(a+b)(a^2 - ab + b^2)	
a^3 - b^3	(a-b)(a^2 + ab + b^2)	
a^2 - b^2	(a+b)(a-b)	*Difference of squares*
(a+b+c)^2	a^2+b^2+c^2 + 2(ab+bc+ca)	
a^3+b^3+c^3 - 3abc	(a+b+c)(a^2+b^2+c^2-ab-bc-ca)	*If a+b+c=0, then a^3+b^3+c^3=3abc*
Quadratic formula	x = [-b ± √(b^2-4ac)] / 2a	*For ax^2 + bx + c = 0*
Discriminant	D = b^2 - 4ac	*D>0: 2 real roots | D=0: 1 root | D<0: no real roots*
Sum of roots	α + β = -b/a	*For quadratic ax^2+bx+c=0*
Product of roots	α × β = c/a	
Form quadratic from roo	x^2 - (sum)x + (product) = 0	
AP: nth term	an = a + (n-1)d	*a = first term, d = common difference*
AP: Sum	Sn = n/2 × [2a + (n-1)d] = n/2 × (a + l)	*l = last term*
AP: 3 terms	Use: a-d, a, a+d (sum = 3a)	*Simplifies equations*
AP: 4 terms	Use: a-3d, a-d, a+d, a+3d (sum = 4a)	
GP: nth term	an = a × r^(n-1)	*r = common ratio*
GP: Sum (finite)	Sn = a(r^n - 1)/(r-1) for r ≠ 1	
GP: Sum (infinite)	S∞ = a/(1-r) for |r| < 1	*Converges only when |r| < 1*
GP: 3 terms	Use: a/r, a, ar (product = a^3)	*Simplifies equations*
Harmonic Mean	HM = 2ab/(a+b)	*Reciprocals form an AP*
AM-GM-HM inequality	AM ≥ GM ≥ HM (equality when all equal)	
AM-GM	(a+b)/2 ≥ √(ab) equality iff a=b	*Used to find min/max*
Cauchy-Schwarz	(a^2+b^2)(c^2+d^2) ≥ (ac+bd)^2	
Log: Product	log(mn) = log m + log n	
Log: Quotient	log(m/n) = log m - log n	
Log: Power	log(m^n) = n·log m	
Log: Base change	log_a(b) = log(b)/log(a) = 1/log_b(a)	
Log: Special values	log 1 = 0 | log_a(a) = 1 | a^(log_a x) = x	
Modulus inequality	|x| < a → -a < x < a	
Modulus equation	|x-a| = b → x = a+b or x = a-b	
Inequalities	Flip sign when multiplying/dividing by negative	
Linear equations	ax + by = c has infinite solutions unless system i	
Cramer's Rule	x = Dx/D, y = Dy/D (for 2×2 system)	*D = determinant of coefficient matrix*

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<tr><td>Triangle — Heron's</td><td>Area = √[s(s-a)(s-b)(s-c)]   where s=(a+b+c)/2</td><td>When all 3 sides known</td></tr>
<tr><td>Equilateral triangle</td><td>Area = (√3/4)a^2 | Height = (√3/2)a</td><td>a = side length</td></tr>
<tr><td>Pythagoras</td><td>a^2 + b^2 = c^2   (c = hypotenuse)</td><td></td></tr>
<tr><td>Pythagorean triples</td><td>3-4-5 | 5-12-13 | 8-15-17 | 7-24-25</td><td>Multiples also valid: 6-8-10, 9-12-15</td></tr>
<tr><td>Angle bisector theorem</td><td>BD/DC = AB/AC</td><td>Bisector of angle A divides BC in ratio AB:AC</td></tr>
<tr><td>Similar triangles</td><td>Ratio of areas = (Ratio of sides)^2</td><td></td></tr>
<tr><td>Midpoint theorem</td><td>Line joining midpoints of 2 sides is parallel to 3</td><td></td></tr>
<tr><td>Basic Proportionality</td><td>If DE ■ BC, then AD/DB = AE/EC</td><td>Thales' theorem</td></tr>
<tr><td>Circles — Area</td><td>Area = π·r^2</td><td></td></tr>
<tr><td>Circles — Circumferenc</td><td>C = 2π·r = π·d</td><td></td></tr>
<tr><td>Circles — Arc length</td><td>L = r·θ (θ in radians) | L = (θ/360)·2πr (degr</td><td></td></tr>
<tr><td>Circles — Sector area</td><td>A = ½r^2·θ (rad) | A = (θ/360)·πr^2 (deg)</td><td></td></tr>
<tr><td>Circles — Chord length</td><td>L = 2r·sin(θ/2)</td><td>θ = central angle</td></tr>
<tr><td>Tangent</td><td>Tangent^2 = d^2 - r^2   (d = distance from externa</td><td>Tangent is perpendicular to radius at point of contact</td></tr>
<tr><td>Tangent-chord angle</td><td>Angle = half the intercepted arc</td><td></td></tr>
<tr><td>Inscribed angle</td><td>Inscribed angle = half the central angle</td><td>Both subtended by same arc</td></tr>
<tr><td>Cyclic quadrilateral</td><td>Opposite angles sum to 180°</td><td></td></tr>
<tr><td>Rectangle</td><td>Area = l×b | Diagonal = √(l^2+b^2) | Perim</td><td></td></tr>
<tr><td>Square</td><td>Area = a^2 | Diagonal = a√2 | Perimeter =</td><td></td></tr>
<tr><td>Parallelogram</td><td>Area = base × height | Diagonals bisect each o</td><td></td></tr>
<tr><td>Trapezium</td><td>Area = ½(a+b)×h   (a,b = parallel sides)</td><td></td></tr>
<tr><td>Rhombus</td><td>Area = ½×d1×d2 | d1^2+d2^2 = 4×side^2</td><td>Diagonals bisect each other at 90°</td></tr>
<tr><td>Regular hexagon</td><td>Area = (3√3/2)a^2 | Perimeter = 6a</td><td>a = side = radius of circumscribed circle</td></tr>
<tr><td>Cylinder</td><td>V = π·r^2·h | CSA = 2π·r·h | TSA = 2π·r(h+</td><td></td></tr>
<tr><td>Cone</td><td>V = ■π·r^2·h | l = √(r^2+h^2) | CSA = π·r·</td><td>l = slant height</td></tr>
<tr><td>Sphere</td><td>V = (4/3)π·r^3 | SA = 4π·r^2</td><td></td></tr>
<tr><td>Hemisphere</td><td>V = (2/3)π·r^3 | CSA = 2π·r^2 | TSA = 3π·r</td><td></td></tr>
<tr><td>Frustum of cone</td><td>V = (π·h/3)(R^2+r^2+Rr) | l = √(h^2+(R-r)^2)</td><td>R = large radius, r = small radius</td></tr>
<tr><td>Cube</td><td>V = a^3 | SA = 6a^2 | Diagonal = a√3</td><td></td></tr>
<tr><td>Cuboid</td><td>V = l·b·h | SA = 2(lb+bh+lh) | Diag = √(l^</td><td></td></tr>
<tr><td>Coordinate — Distance</td><td>D = √[(x2-x1)^2 + (y2-y1)^2]</td><td></td></tr>
<tr><td>Coordinate — Midpoint</td><td>((x1+x2)/2, (y1+y2)/2)</td><td></td></tr>
<tr><td>Coordinate — Section</td><td>(m·x2+n·x1)/(m+n), (m·y2+n·y1)/(m+n)</td><td>Divides line in ratio m:n internally</td></tr>
<tr><td>Slope</td><td>m = (y2-y1)/(x2-x1) | y = mx+c</td><td></td></tr>
<tr><td>Perpendicular slopes</td><td>m1 × m2 = -1</td><td>Product of slopes of perpendicular lines = -1</td></tr>
<tr><td>Parallel slopes</td><td>m1 = m2</td><td>Parallel lines have equal slopes</td></tr>
<tr><td>Distance point-to-line</td><td>d = |ax1+by1+c| / √(a^2+b^2)</td><td>For line ax+by+c=0 and point (x1,y1)</td></tr>
<tr><td>Centroid of triangle</td><td>((x1+x2+x3)/3, (y1+y2+y3)/3)</td><td></td></tr>
<tr><td>Area of triangle (coord)</td><td>½|x1(y2-y3) + x2(y3-y1) + x3(y1-y2)|</td><td></td></tr>
</table>

PERMUTATION & COMBINATION	FORMULA	NOTE
Factorial	n! = n×(n-1)×...×2×1 | 0! = 1 | 1! = 1	
nPr — Permutation	nPr = n!/(n-r)! = n×(n-1)×...×(n-r+1)	*Ordered arrangement of r from n*
nCr — Combination	nCr = n!/[r!(n-r)!] | nCr = nC(n-r)	*Unordered selection of r from n*
Pascal's identity	nCr = (n-1)C(r-1) + (n-1)Cr	
nC0 = nCn = 1	nC1 = nC(n-1) = n	
Sum of all nCr	nC0 + nC1 + ... + nCn = 2^n	
Circular arrangement	(n-1)! ways	*Fix one element to remove rotational symmetry*
Necklace/bracelet	(n-1)!/2	*Further divide by 2 for flip symmetry*
Identical objects	n objects with p identical: n!/p!	*Divide by factorials of repeated counts*
Multinomial	n!/(p!·q!·r!) for n=p+q+r	
Distribution (distinct)	n objects into r distinct boxes = r^n	*Each object has r choices*
Derangement	D(n) = n![1 - 1/1! + 1/2! - 1/3! + ... + (-1)^n/n!	*No object in its original position*
Stars & Bars	n identical into r distinct boxes = C(n+r-1, r-1)	*Non-negative integer solutions to x1+x2+...+xr=n*
Binomial theorem	(a+b)^n = Σ C(n,r)·a^(n-r)·b^r r=0 to n	
General term	T(r+1) = C(n,r)·a^(n-r)·b^r	*In binomial expansion of (a+b)^n*
Middle term	If n even: T(n/2+1) is middle term	*If n odd: T((n+1)/2) and T((n+3)/2) are middle terms*
Selecting from groups	m from group A, n from group B = C(a,m)×C(b,n)	
Atleast/Atmost	Use complementary counting: total - opposite	

PROBABILITY	FORMULA	NOTE
Basic	P(A) = Favourable outcomes / Total outcomes |	
Complement	P(A') = 1 - P(A) | P(A) + P(A') = 1	
Addition rule	P(A∪B) = P(A) + P(B) - P(A∩B)	
Mutually exclusive	P(A∪B) = P(A) + P(B) [if A∩B = ∅]	
Exhaustive events	P(A) + P(B) + ... = 1	
Independent events	P(A∩B) = P(A) × P(B)	*Events do not affect each other*
Conditional probability	P(A|B) = P(A∩B) / P(B)	*Probability of A given B has occurred*
Multiplication rule	P(A∩B) = P(A) × P(B|A) = P(B) × P(A|B)	
Bayes' theorem	P(A|B) = P(B|A)·P(A) / P(B)	*For updating probabilities with new evidence*
Total probability	P(B) = Σ P(B|Ai)·P(Ai)	*Sum over all mutually exclusive, exhaustive Ai*
At least one	P(at least one) = 1 - P(none)	*Complement is often easier*
Coins	P(r heads in n tosses) = C(n,r) × (1/2)^n	
Dice	P(sum=s) varies | Use systematic counting	*Total outcomes = 6^n for n dice*

SEQUENCES, SERIES & SPECIAL	FORMULA	NOTE
Sum of natural numbers	Sn = n(n+1)/2	*1+2+3+...+n*
Sum of squares	Sn = n(n+1)(2n+1)/6	*1^2+2^2+...+n^2*
Sum of cubes	Sn = [n(n+1)/2]^2	*1^3+2^3+...+n^3 = (Sn of naturals)^2*
Infinite GP sum	S = a/(1-r) for |r| < 1	
Special products	n(n+1)(n+2)/6 = C(n+2,3)	
Fibonacci	1,1,2,3,5,8,13,21,34,55... | F(n) = F(n-1)+F(n-2	*Golden ratio ϕ = (1+√5)/2 ≈ 1.618*
AP-GP combined	Sum of AGP: use formula or differentiation trick	

CLOCKS & CALENDARS	FORMULA	NOTE
Clock angle	Angle = |30H - 5.5M| degrees	H = hours, M = minutes
Hands coincide	Every 65 5/11 minutes | 11 times in 12 hours	Not at 12-hr mark exactly
Hands opposite (180°)	Every 65 5/11 minutes | 11 times in 12 hours	Offset by half cycle from coincide times
Hands at 90°	22 times in 12 hours	
Leap year rule	Div by 4, except centuries need div by 400	2000 = leap, 1900 = not leap
Odd days	Ordinary year = 1 odd day | Leap year = 2 odd	
Century odd days	100 yrs = 5 | 200 yrs = 3 | 300 yrs = 1	
Month odd days	Jan=3, Feb=0(28)/1(29), Mar=3, Apr=2, May=3, Jun=1	
Month odd days (2)	Jul=3, Aug=3, Sep=2, Oct=3, Nov=2, Dec=3	
Day code	0=Sun, 1=Mon, 2=Tue, 3=Wed, 4=Thu, 5=Fri, 6=Sat	

MENSURATION — QUICK REFERENCE	FORMULA	NOTE
2D: Square	Area=a^2 | Perimeter=4a | Diagonal=a√2	
2D: Rectangle	Area=lb | Perimeter=2(l+b) | Diagonal=√(l^2+b^	
2D: Triangle	Area=½bh | Perimeter=a+b+c	
2D: Equilateral ■	Area=(√3/4)a^2 | Height=(√3/2)a	
2D: Circle	Area=πr^2 | Circumference=2πr	
2D: Semicircle	Area=πr^2/2 | Perimeter=πr+2r	
2D: Sector	Area=θr^2/2 (rad) | Arc=rθ	
2D: Rhombus	Area=½d1d2 | Perimeter=4a	a=√((d1/2)^2+(d2/2)^2)
2D: Trapezium	Area=½(a+b)h | Perimeter=a+b+c+d	
2D: Parallelogram	Area=bh | Perimeter=2(a+b)	
3D: Cube	V=a^3 | TSA=6a^2 | Diagonal=a√3	
3D: Cuboid	V=lbh | TSA=2(lb+bh+lh) | Diag=√(l^2+b^2+h^2)	
3D: Cylinder	V=πr^2h | CSA=2πrh | TSA=2πr(h+r)	
3D: Cone	V=πr^2h/3 | CSA=πrl | TSA=πr(l+r) | l=√(r^2+	
3D: Sphere	V=4πr^3/3 | SA=4πr^2	
3D: Hemisphere	V=2πr^3/3 | CSA=2πr^2 | TSA=3πr^2	
3D: Frustum	V=πh(R^2+r^2+Rr)/3 | CSA=π(R+r)l | l=√(h^2+(R-	